Ramsey goodness of bounded degree trees

Igor Balla

ETH Zurich

Given a pair of graphs G and H, the Ramsey number $R(G,H)$ is the smallest N such that every red-blue coloring of the edges of the complete graph K_N contains a red copy of G or a blue copy of H. If a graph G is connected, it is well known and easy to show that $R(G,H) \geq (|G|-1)(\chi(H)-1)+\sigma(H)$, where $\chi(H)$ is the chromatic number of H and $\sigma(H)$ is the size of the smallest color class in a $\chi(H)$-coloring of H. A graph G is called H-good if $R(G,H) = (|G|-1)(\chi(H)-1)+\sigma(H)$. The notion of Ramsey goodness was introduced by Burr and Erdős in 1983 and has been extensively studied since then. In this paper we show that if $n \geq \Omega(|H| \log^4 |H|)$ then every n-vertex bounded degree tree T is H-good. The dependency between n and $|H|$ is tight up to log factors. This substantially improves a result of Erdős, Faudree, Rousseau, and Schelp from 1985, who proved that n-vertex bounded degree trees are H-good when when $n \geq \Omega(|H|^4)$.