The edge isoperimetric problem for a graph $G$ is to determine, for each $n$, the minimum number of edges leaving any set of $n$ vertices. Exact solutions are known only in special cases, for example when $G$ is the usual integer lattice. The most natural open problem was to answer this question for the ‘strong lattice’, with edges between points at $l_\infty$ distance 1. Whilst studying this question we in fact solved the edge isoperimetric problem asymptotically for every Cayley graph on $\mathbb{Z}^d$. I’ll talk about how to go from the specification of a lattice to a corresponding near-optimal shape, for both this and the related vertex isoperimetric problem. Joint work with Joshua Erde.