THE ASYMPTOTIC COMPLEXITY OF MATRIX REDUCTION OVER FINITE FIELDS

DEMETRES CHRISTOFOIDES

ABSTRACT. Consider an invertible $n \times n$ matrix over some field. The Gauss-Jordan elimination reduces this matrix to the identity matrix using at most $n^2$ row operations and in general that many operations might be needed.

In [1] the authors considered matrices in $\text{GL}(n, q)$, the set of $n \times n$ invertible matrices in the finite field of $q$ elements, and provided an algorithm using only row operations which performs asymptotically better than the Gauss-Jordan elimination. More specifically their ‘striped elimination algorithm’ has asymptotic complexity $\frac{n^2 \log q}{n}$. Furthermore they proved that up to a constant factor this algorithm is best possible as almost all matrices in $\text{GL}(n, q)$ need asymptotically at least $\frac{n^2}{2 \log_q n}$ operations.

In this talk we will show that the ‘striped elimination algorithm’ is asymptotically optimal by proving that almost all matrices in $\text{GL}(n, q)$ need asymptotically at least $\frac{n^2}{\log_q n}$ operations.

REFERENCES