Core forging in random graphs

Oliver Cooley

Graz University of Technology

The problem of analysing (e.g. determining the order and size of) the k-core of a random graph $G(n, d/n)$ with fixed average degree d has received a significant amount of attention in recent years, including several proofs of the well-known phase transition result, that there is a threshold d_k for d at which the k-core becomes non-empty with high probability.

Previous studies of this problem have almost universally been based on an analysis of the "peeling process" by which the k-core can be determined. We present a different approach: we introduce a random algorithm which constructs a graph for which we know in advance which vertices will be in the core and which will not, as well as how these vertices interact with each other. Furthermore, the asymptotic distribution of the resulting random graph is identical to $G(n, d/n)$.

The proof is inspired by a message-passing algorithm known as Warning Propagation, which was introduced in non-rigorous work by physicists. As a corollary of our analysis, we also obtain a new proof of the phase transition result for the existence of a non-empty k-core.

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