Families of Mass Destruction

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Set shattering is a much-studied area of extremal set theory, with many applications to theoretical computer science, probability theory and beyond. We say that a family of sets $\mathcal{F}$ over the ground set $[n]$ shatters a set $A \subseteq [n]$ if its members intersect A in every possible way; that is, $\{F \cap A : F \in \mathcal{F}\} = 2^A$. One problem dating back to the 1970's, of both theoretical and practical interest, asks for the smallest possible (n, k)-universal family, which is a set family that shatters every k-set over the n-element ground set.

We introduce a refined version of this question, asking how many k-sets a family of size m can shatter, resolving the first case of interest. We show that as soon as the family is large enough to shatter a single k-set, it can shatter a positive proportion of all k-sets, and determine the maximum possible number of shattered k-sets. Our construction generalises to the related setting of covering arrays, studied in combinatorial design theory.

This is joint work with Tamás Mészáros.