Improved bounds for sampling colorings of sparse random graphs

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We study the mixing properties of the single-site Markov chain known as the Glauber dynamics for sampling $k$-colorings of a sparse random graph $G(n, d/n)$ for constant $d$. The best known rapid mixing results for general graphs are in terms of the maximum degree $\Delta$ of the input graph $G$ and hold when $k > 11\Delta/6$ for all $G$. Improved results hold when $k > \alpha \Delta$ for graphs with girth $\geq 5$ and $\Delta$ sufficiently large where $\alpha \approx 1.7632\ldots$ is the root of $\alpha = \exp(1/\alpha)$; further improvements on the constant $\alpha$ hold with stronger girth and maximum degree assumptions.

For sparse random graphs the maximum degree is a function of $n$ and the goal is to obtain results in terms of the expected degree $d$. The following rapid mixing results for $G(n, d/n)$ hold with high probability over the choice of the random graph for sufficiently large constant $d$. Mossel and Sly (2009) proved rapid mixing for constant $k$, and Efthymiou (2014) improved this to $k$ linear in $d$. The condition was improved to $k > 3d$ by Yin and Zhang (2016) using non-MCMC methods.

Here we prove rapid mixing when $k > \alpha d$ where $\alpha \approx 1.7632\ldots$ is the same constant as above. Moreover we obtain $O(n^3)$ mixing time of the Glauber dynamics, while in previous rapid mixing results the exponent was an increasing function in $d$. As in previous results for random graphs our proof analyzes an appropriately defined block dynamics to “hide” high-degree vertices. One new aspect in our improved approach is utilizing so-called local uniformity properties for the analysis of block dynamics. To analyze the “burn-in” phase we prove a concentration inequality for the number of disagreements propagating in large blocks.

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