Infinite size-biased orders and permutations

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The uniform permutation of the finite set $[n]$ has a natural infinite analogue, which is a random order on $\mathbb{N}$. In this talk we extend this idea to non-uniform permutations. For a summable collection of weights $(w_i)$ a size-biased permutation of $\mathbb{N}$ is defined via successive sampling without replacement, where each yet unsampled element $i$ is drawn with probability proportional to $w_i$. When the weights are not summable this procedure is meaningless. Instead, it is possible to define a consistent system of finite size-biased permutations which altogether yield an infinite random order. We give constructions of the infinite size-biased orders and a complete classification of their possible types, relating them to density properties of a non-homogeneous sequence of exponential random variables.