Tilings of product spaces

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A Tetris tile is a connected subset of $\mathbb{Z}^2$ of size 4. More generally, we define an n-dimensional shape to be any (non-empty) finite subset of $\mathbb{Z}^n$. Does a given shape $T \subset \mathbb{Z}^n$ tile the n-dimensional space, meaning that $\mathbb{Z}^n$ can be partitioned into copies of T? Of course, some shapes tile $\mathbb{Z}^n$ and some do not. Moreover, some shapes that do not tile $\mathbb{Z}^n$ do tile $\mathbb{Z}^{n+1}$. Chalcraft conjectured that every shape in $\mathbb{Z}^n$ tiles $\mathbb{Z}^d$ for some $d \geq n$. We prove this conjecture and examine related questions regarding posets and the hypercube graph Q_n.

This talk is based on joint work with Leader, Letzter, Tan and Tomon.