The chromatic number of dense random graphs

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Determining the chromatic number of the random graph $G(n, p)$ is one of the classic challenges in random graph theory. A celebrated breakthrough by Bollobás in 1987 first established the asymptotic value of the chromatic number of $G(n, \frac{1}{2})$.

In this talk, new upper and lower bounds for the chromatic number of the dense random graph $G(n, p)$ with p constant are established. These bounds are the first that match each other up to a term of size $o(1)$ in the denominator, and they determine the average colour class size in an optimal colouring up to an additive term of size $o(1)$, answering a question of Kang and McDiarmid. Somewhat surprisingly, the behaviour of the chromatic number changes at $p = 1 - 1/e^2 \approx 0.86$, with a different limiting effect being dominant below and above this value.

We also discuss the equitable chromatic number of the dense random graph $G(n, m)$, which is concentrated on just one value on a subsequence of the integers.