Ramsey graphs induce subgraphs of many different sizes

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1 Abstract

A graph on $n$ vertices is said to be $C$-Ramsey if every clique and independent set of the graph has size at most $C \log n$. The only known constructions of Ramsey graphs are probabilistic in nature and it is believed that such graphs possess many of the same properties as dense random graphs. A conjecture of Erdős and McKay addresses such a property; they ask whether there exists a constant $\gamma = \gamma(C)$ such that every $C$-Ramsey graph on $n$ vertices contains an induced subgraph with $k$ edges for any $k \in \{0, \ldots, \gamma n^2\}$.

We make the following small step towards the solution of this conjecture: we prove that for any fixed $C > 0$, if $G$ is a $C$-Ramsey graph on $n$ vertices, then there are at least $n^2 - o(1)$ different positive integers $k$ that occur as the number of edges of some induced subgraph of $G$.

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